Multiple Goods, Consumer Heterogeneity and Revealed Preference

Richard Blundell(UCL), Dennis Kristensen(UCL) and Rosa Matzkin(UCLA)

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This talk builds on three related papers:

- Blundell, Kristensen, Matzkin (2011a) "Bounding Quantile Demand Functions Using Revealed Preference Inequalities"

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- ▶ Focus here is on identification and estimation when there are many heterogeneous consumers, a finite number of markets (prices) and non-additive heterogeneity.

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- Typically dealing with a finite number of markets (prices) and many (heterogeneous) consumers.

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- FOC - system of simultaneous equations with nonadditive unobservables

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- For each price regime the d_g are *expansion paths* (or Engel curves) for each heterogeneous consumer of type $[\varepsilon, \mathbf{z}]$
- Key assumptions will pertain to the dimension and direction of unobserved heterogeneity ε, and to the specification of observed heterogeneity $\mathbf{z}$.

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$$\frac{\partial d\left(p, I, \varepsilon\right)}{\partial \varepsilon}$$

$$= -\frac{w_{10}\left(y_1, \varepsilon\right)}{v_{11}\left(y_1, I - py_1\right) - 2\ v_{10}\left(y_1, I - py_1\right) p + v_{00}\left(y_1, I - py_1\right)\ p^2 + w_{11}\left(y\right.}$$

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- Assuming that ε is distributed independently of (p, I), the demand function is strictly increasing in ε, and F_{ε} is strictly increasing at ε,

$$d(p', I', \varepsilon) - d\left(\widetilde{p}, \widetilde{I}, \varepsilon\right) = F^{-1}_{Y|(p,I)=(p',I')}\left(F_{Y|(p,I)=(\widetilde{p},\widetilde{I})}(y_1)\right) - y_1$$

where y_1 is the observed consumption when budget is $\left(\widetilde{p}, \widetilde{I}\right)$.

Implied Restrictions on Demands

- If consumer ε satisfies Revealed Preference then the inequalities:

$$\widetilde{p}_1 \left(y_1' - \widetilde{y}_1\right) + \widetilde{p}_0(y_0' - \widetilde{y}_0) \leq \widetilde{I} \quad \Rightarrow \quad p_1' \left(y_1' - \widetilde{y}_1\right) + p_0'(y_0' - \widetilde{y}_0) < I'$$

allow us to bound demand on a new budget $\left(\widetilde{p}, \widetilde{I}\right)$ for each consumer ε, where $y_1' = d\left(p', I', \varepsilon\right)$ and $y_0' = \left(I' - p_1' d\left(p', I', \varepsilon\right)\right) / p_0'$.

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- In this paper we show same set identification results hold for each consumer of type $[\varepsilon_1, ..., \varepsilon_G]$ under RP inequality restrictions

Results for infinitessimal changes in prices.

We know

$$\frac{\partial d\left(p, I, \varepsilon\right)}{\partial\left(p, I\right)} = -\left[\frac{\partial F_{Y|(p,I)}\left(d\left(p, I, \varepsilon\right)\right)}{\partial y}\right]^{-1} \frac{\partial F_{Y|(p,I)}\left(d\left(p, I, \varepsilon\right)\right)}{\partial (p, I)}$$

(Matzkin (1999), Chesher (2003)).

- And since each consumer ε satisfies the Integrability Conditions

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- Which allow us to bound the effect of an infinitessimal change in price.

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- Figures of quantile expansion paths, demand bounds and confidence sets in Figures 3 and 4.

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- ▶ As before we assume ε_1 is scalar and c_1 is strictly increasing in ε_1.

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- Likewise p_2, and ε_2, are exclusive to c_2. So that we have:

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- These restricted specifications will be important in our discussion of identification and estimation

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- Suppose preferences are such that $[y_1, y_0]$ form a separable sub-group within $[y_1, y_0, y_2]$. In this case, utility has the recursive form

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- ▶ Can relax preference assumptions to allow ε_1 to enter c_2.
- z_1 (and p_1) is excluded from c_2 and could act an instrument for y_1 in the QCF estimation of c_2, as in Chesher (2003) and Imbens and Newey (2009).

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- Blundell and Matzkin (2010) derive the complete set of *if and only if* conditions for nonseparable simultaneous equations models that generate triangular systems and therefore permit estimation by the control function (QCF) approach.

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- The BM conditions cover preferences that include the conditional recursive separability form above.
- For example,

$$V(\varepsilon_1, \varepsilon_2, y_2) + W(\varepsilon_1, y_1, y_2) + y_0$$

e.g.

$$= (\varepsilon_1 + \varepsilon_2)\ u(y_2) + \varepsilon_1\ \log(y_1 - u(y_2)) + y_0$$

The general G+1>2 case

- If demand functions are invertible in $(\varepsilon_1, ..., \varepsilon_G)$, we can write $(\varepsilon_1, ..., \varepsilon_G)$ as

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- Can use the transformation of variables equation to determine identification (Matzkin (2010))

$$f_{Y|p,I,z}(y) = f_\varepsilon(r(y,p,I,z)) \left| \frac{\partial r(y,p,I,z)}{\partial y} \right|$$

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- As we show, estimation can proceed using the average derivative method of Matzkin (2010).

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- Then, by Gale and Nikaido (1965), the system is invertible: There exist functions $r^1, ..., r^G$ such that

$$\begin{aligned} \varepsilon_1 + z_1 &= r^1(y_1, ..., y_G, p_1, ..., p_K, I) \\ &\cdots \\ \varepsilon_G + z_G &= r^G(y_1, ..., y_G, p_1, ..., p_K, I) \end{aligned}$$

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- Identification of $r \Rightarrow$ identification of h

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- Use mode assumption on ε, to recover the level of r at some value of y.

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- ▶ Figure 5....

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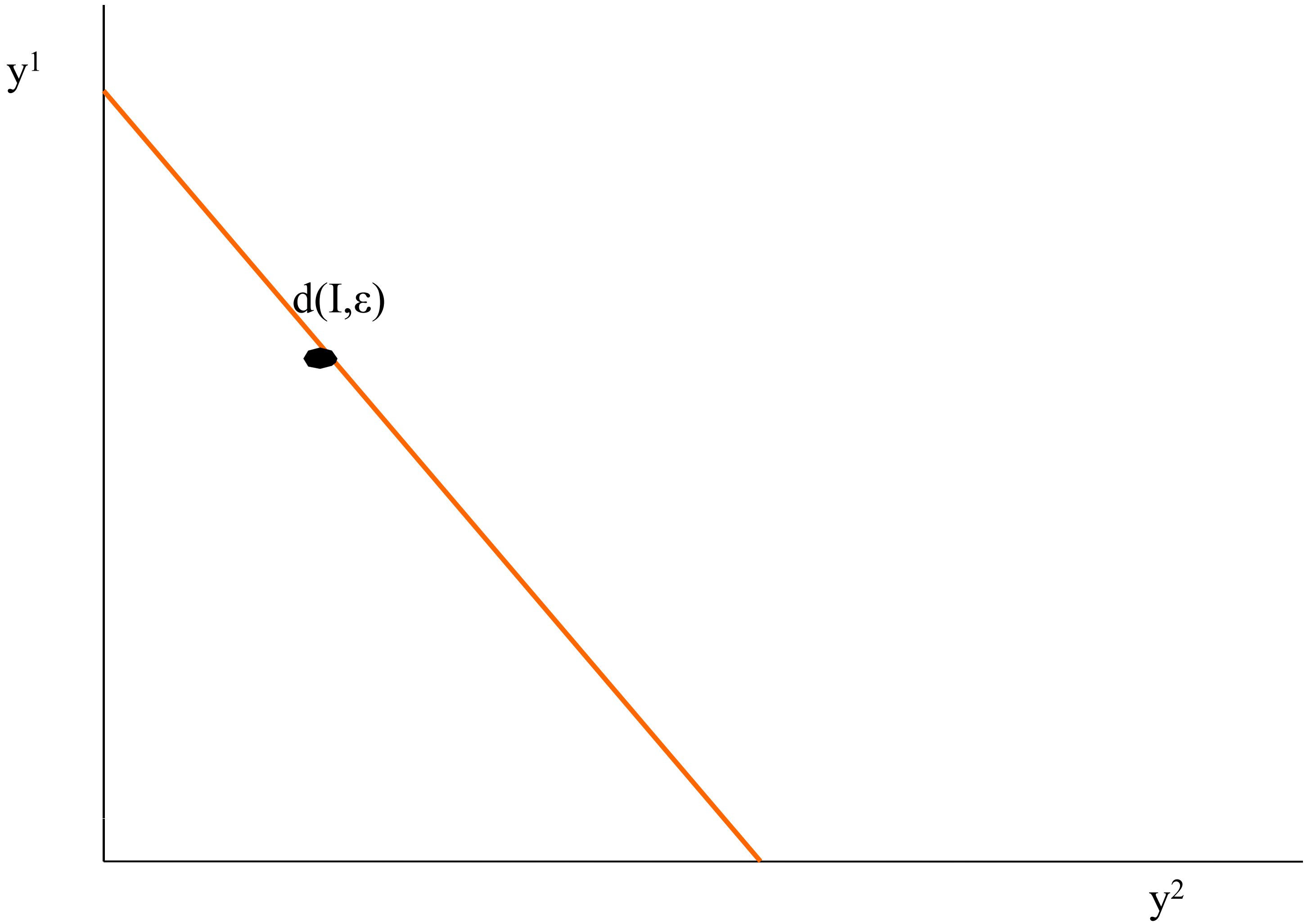
- ▶ Focus on the case of discrete prices (finite markets) and many heterogeneous consumers.

- ▶ Show how to use restrictions implied by revealed preference / integrability to bound the distribution of predicted demand at unobserved prices (policy counterfactual).

Figure 1a: The distribution of demands across consumers indexed by 'ε'



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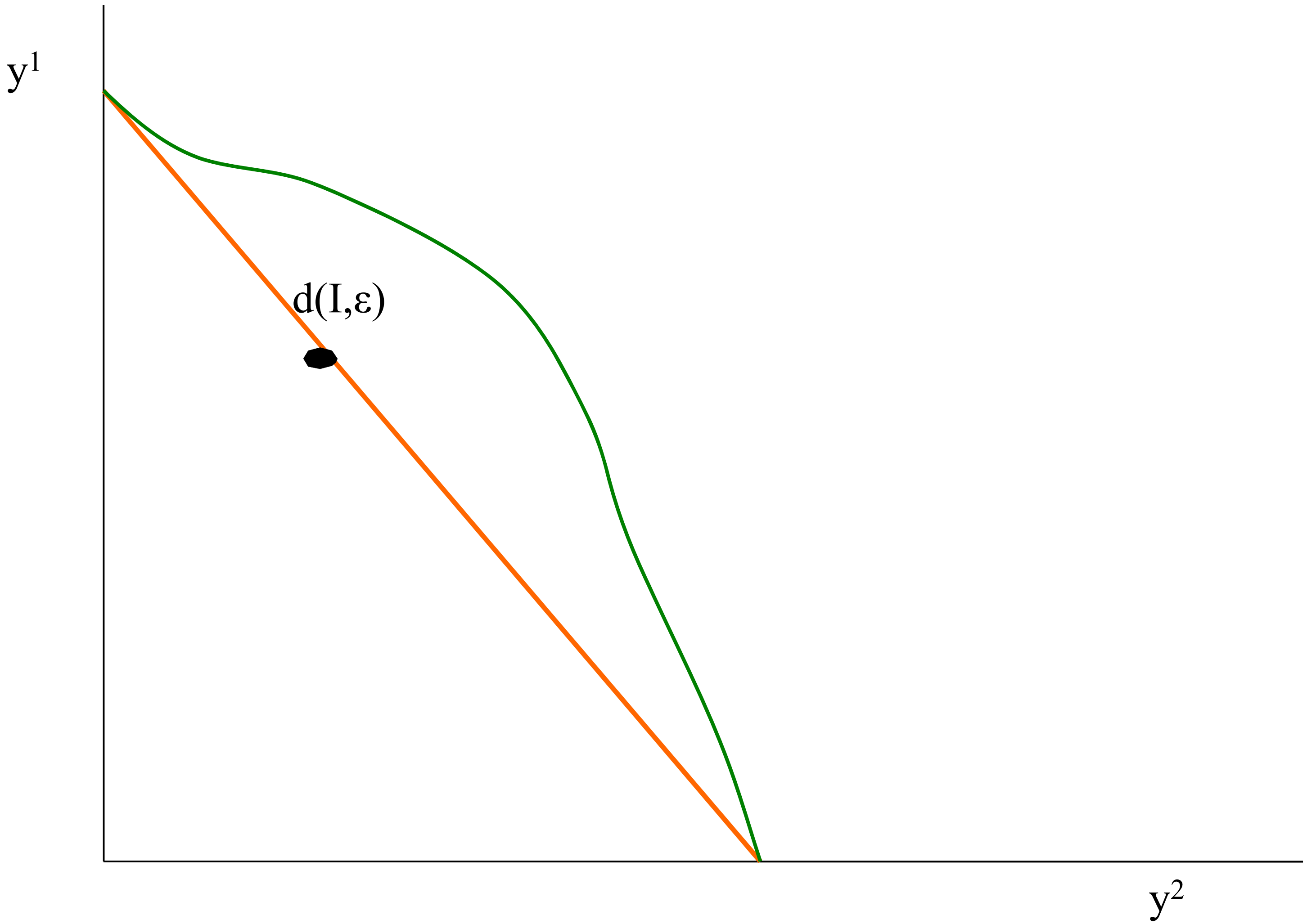


Figure 1b: Monotonicity in ‘ε’ and rank preserving on the budget constraint

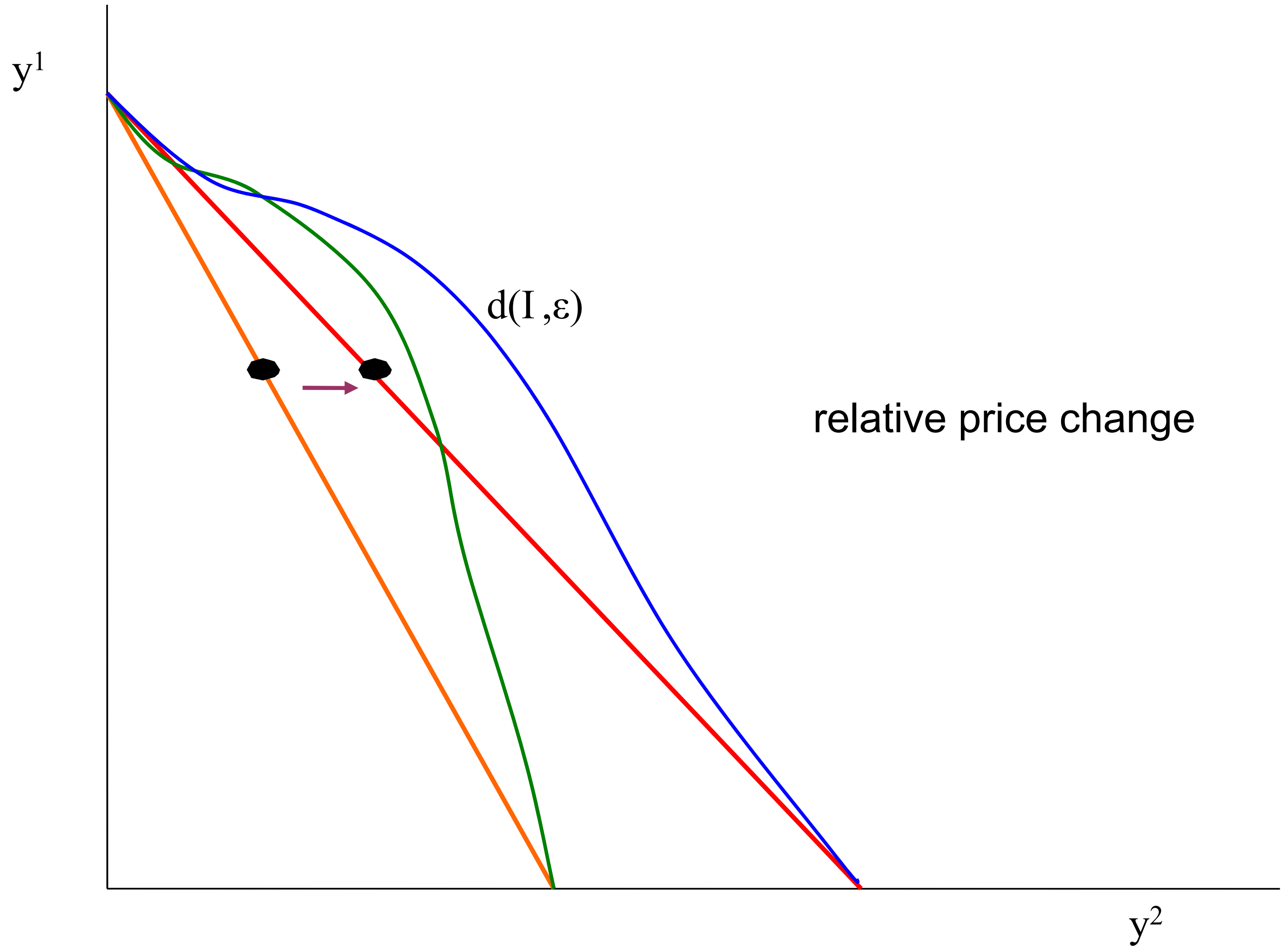


Figure 1c: The quantile expansion path

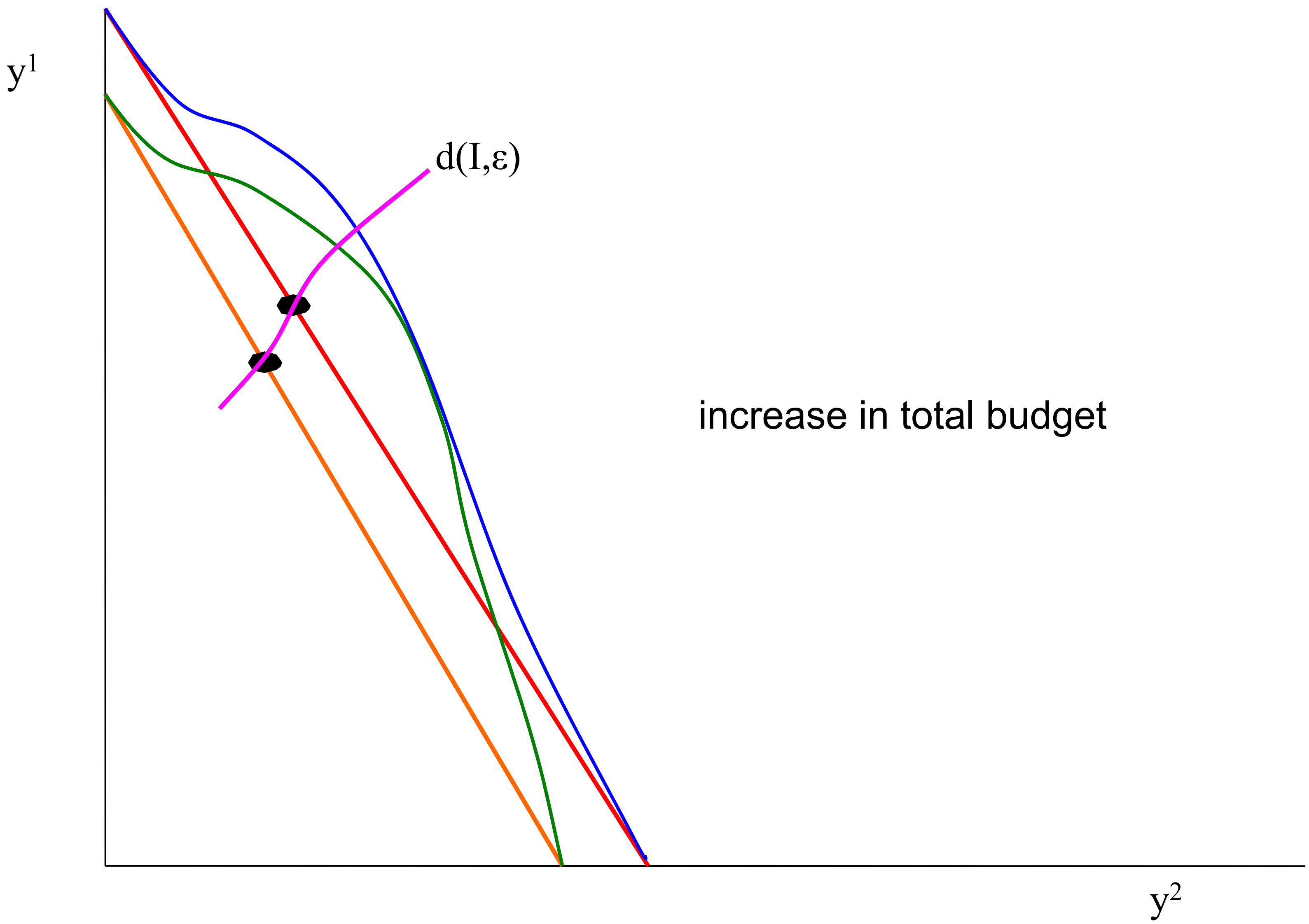
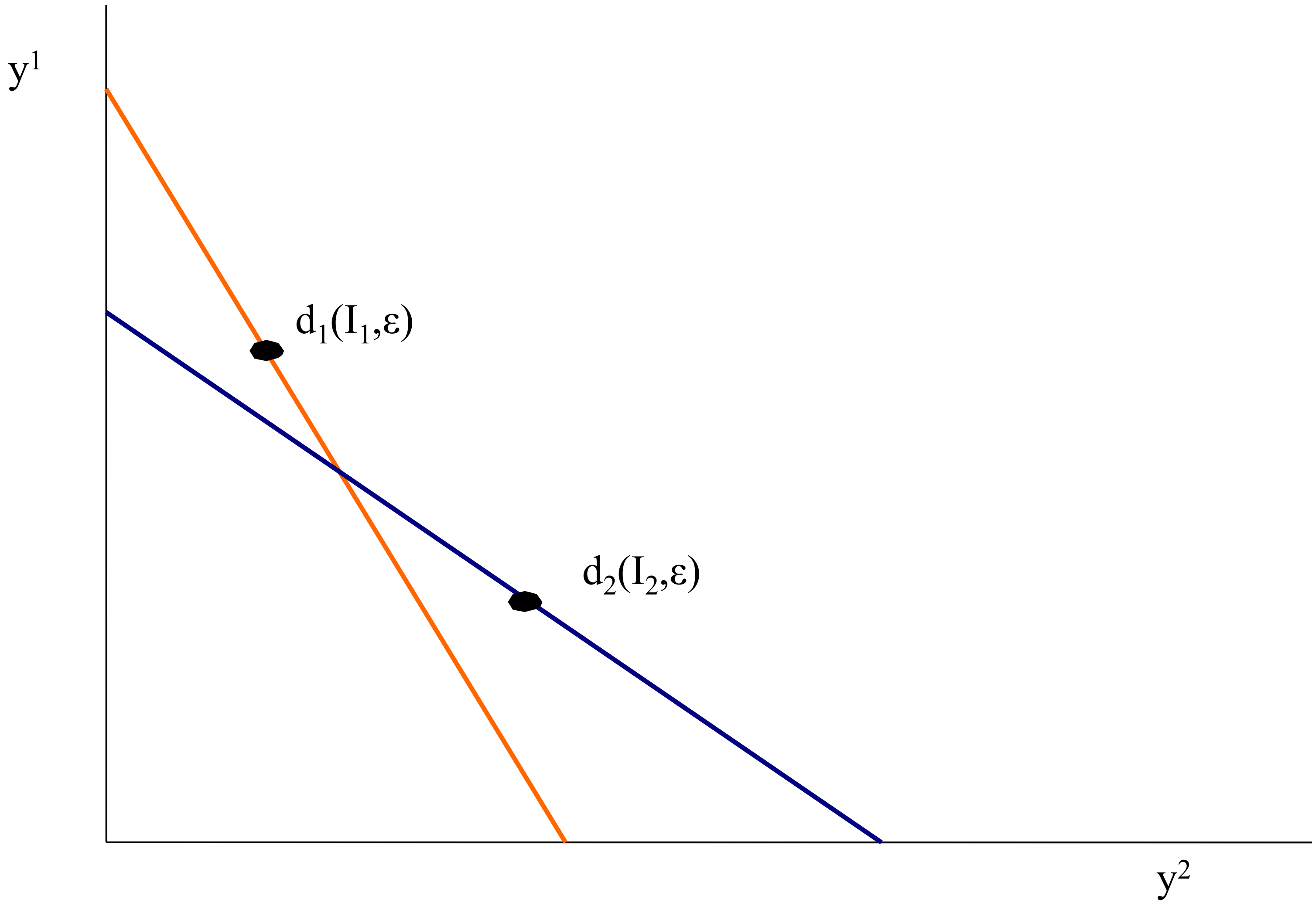


Figure 2a: Generating a Support Set with RP for consumer 'ε'



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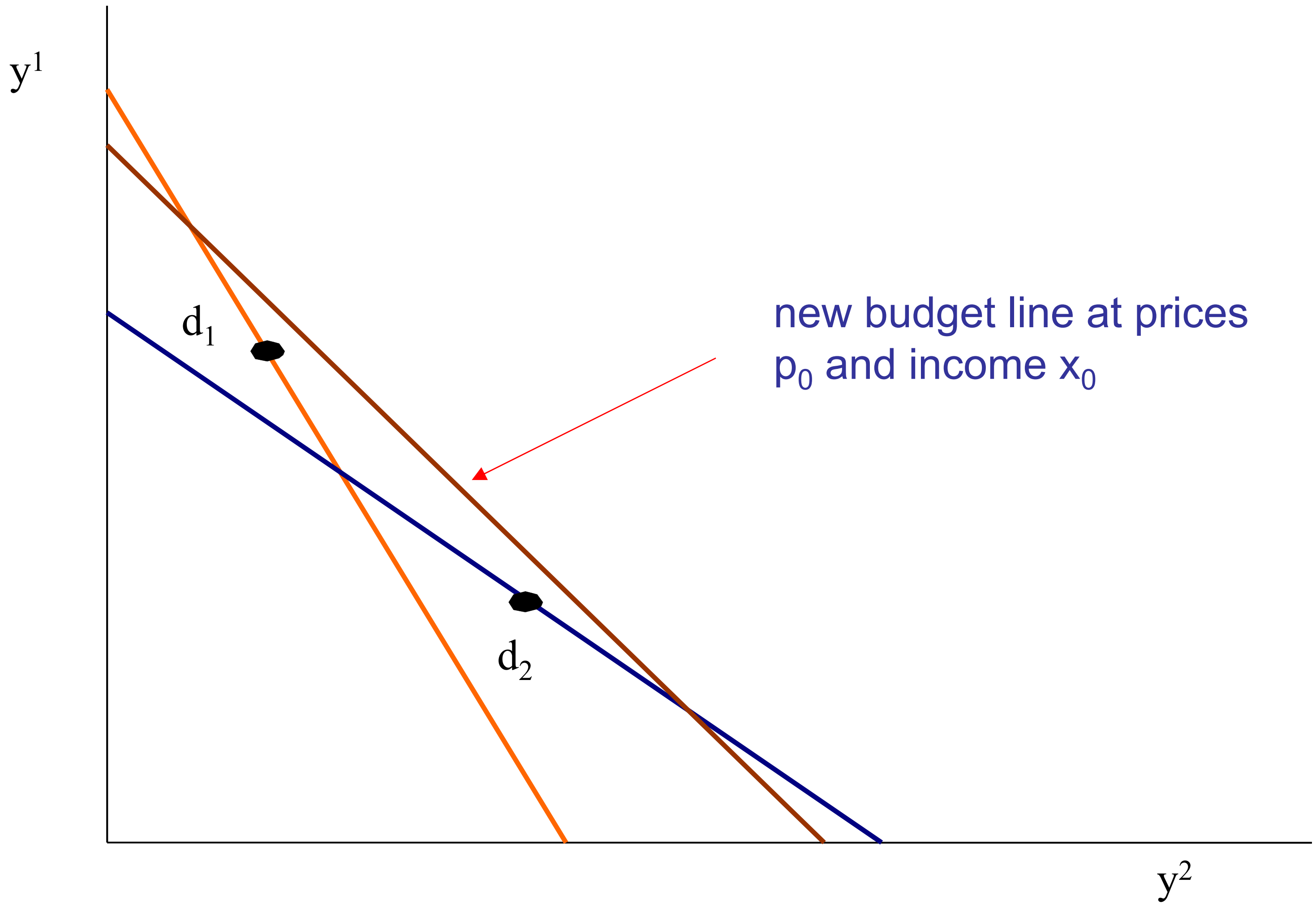


Figure 2a: Generating a Support Set with RP for consumer ‘ε’

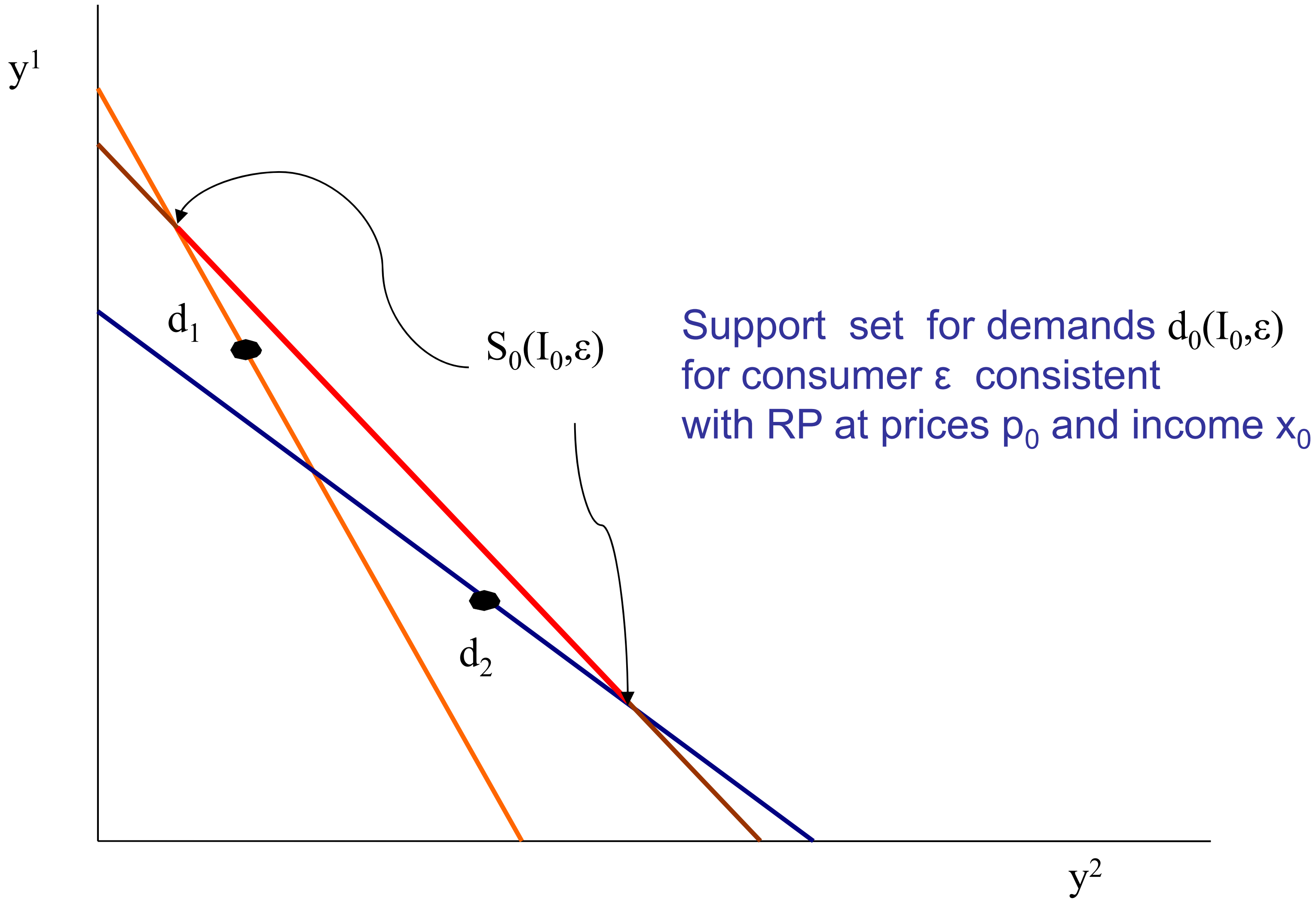


Figure 2d. Improving the support set with *e-bounds,* for consumer ‘ε’

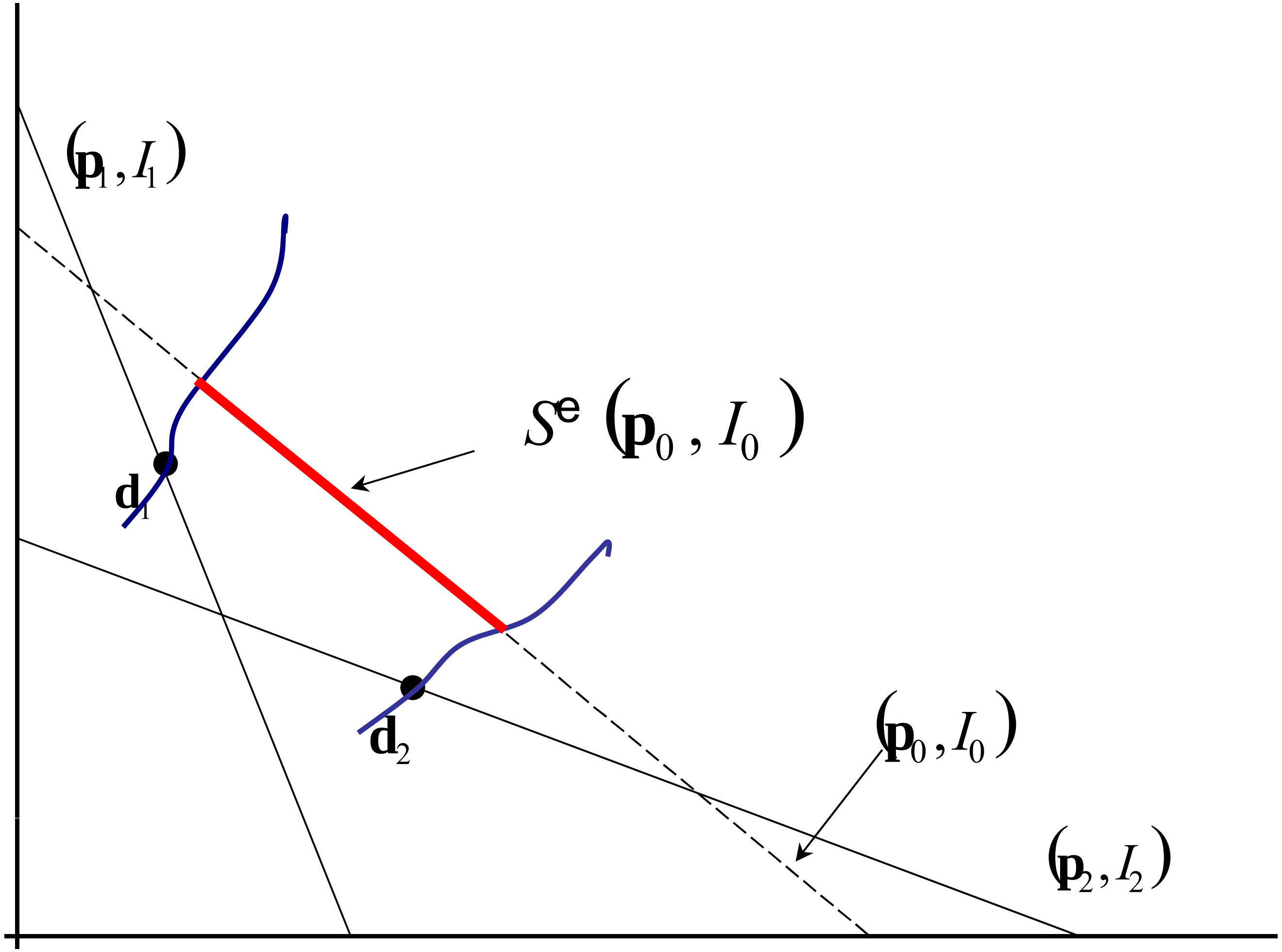


Figure 2e: The best support set with many price regimes

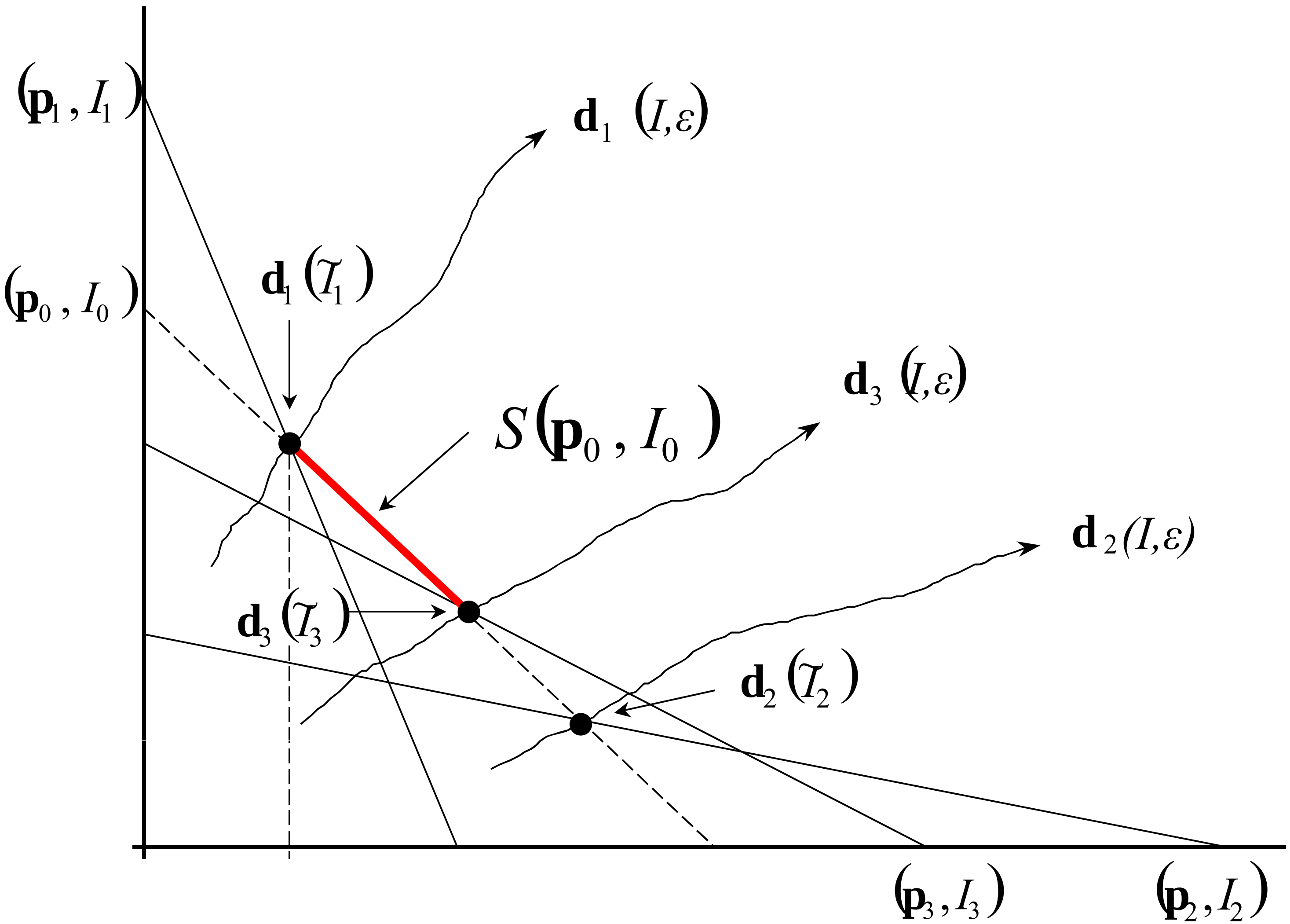
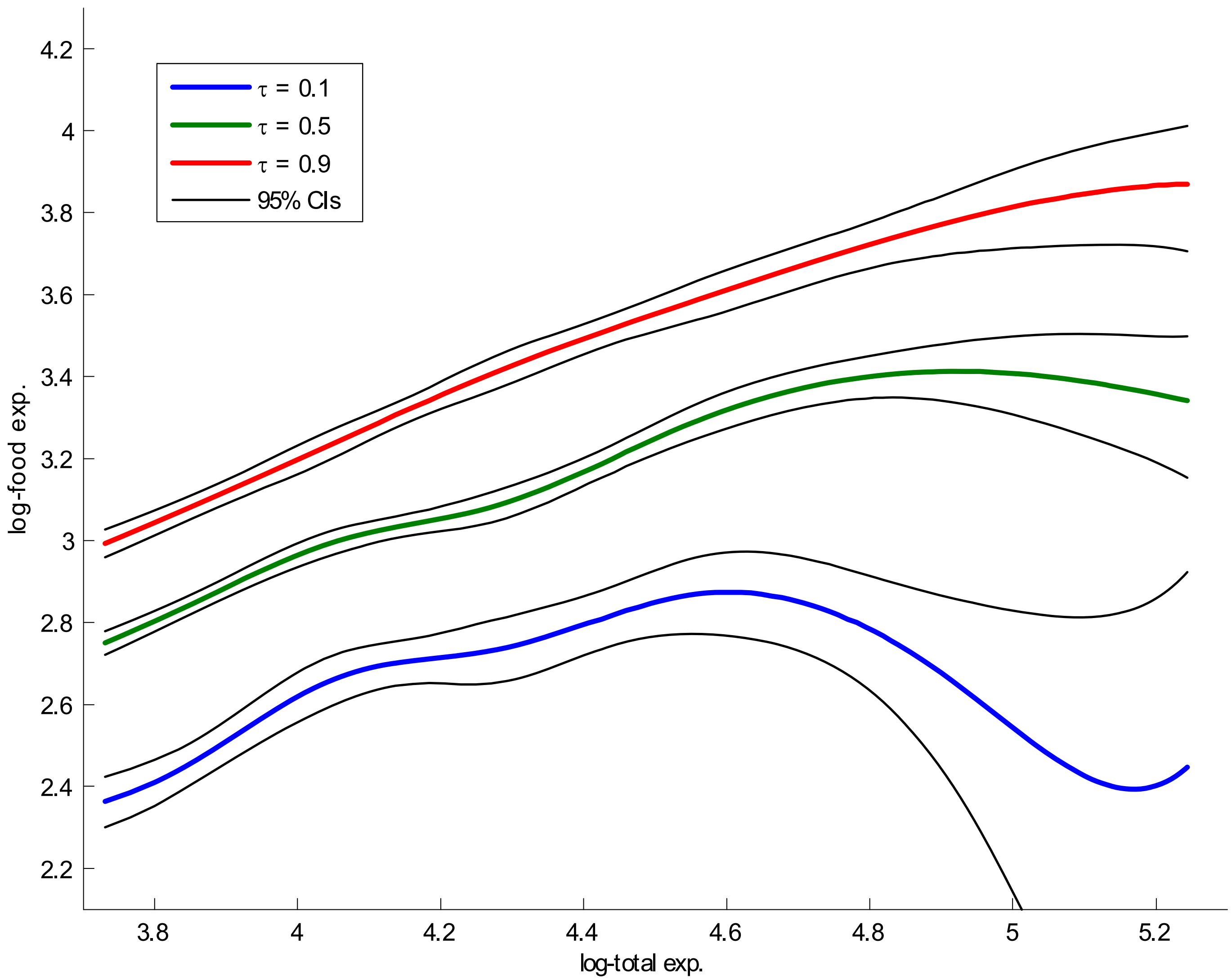
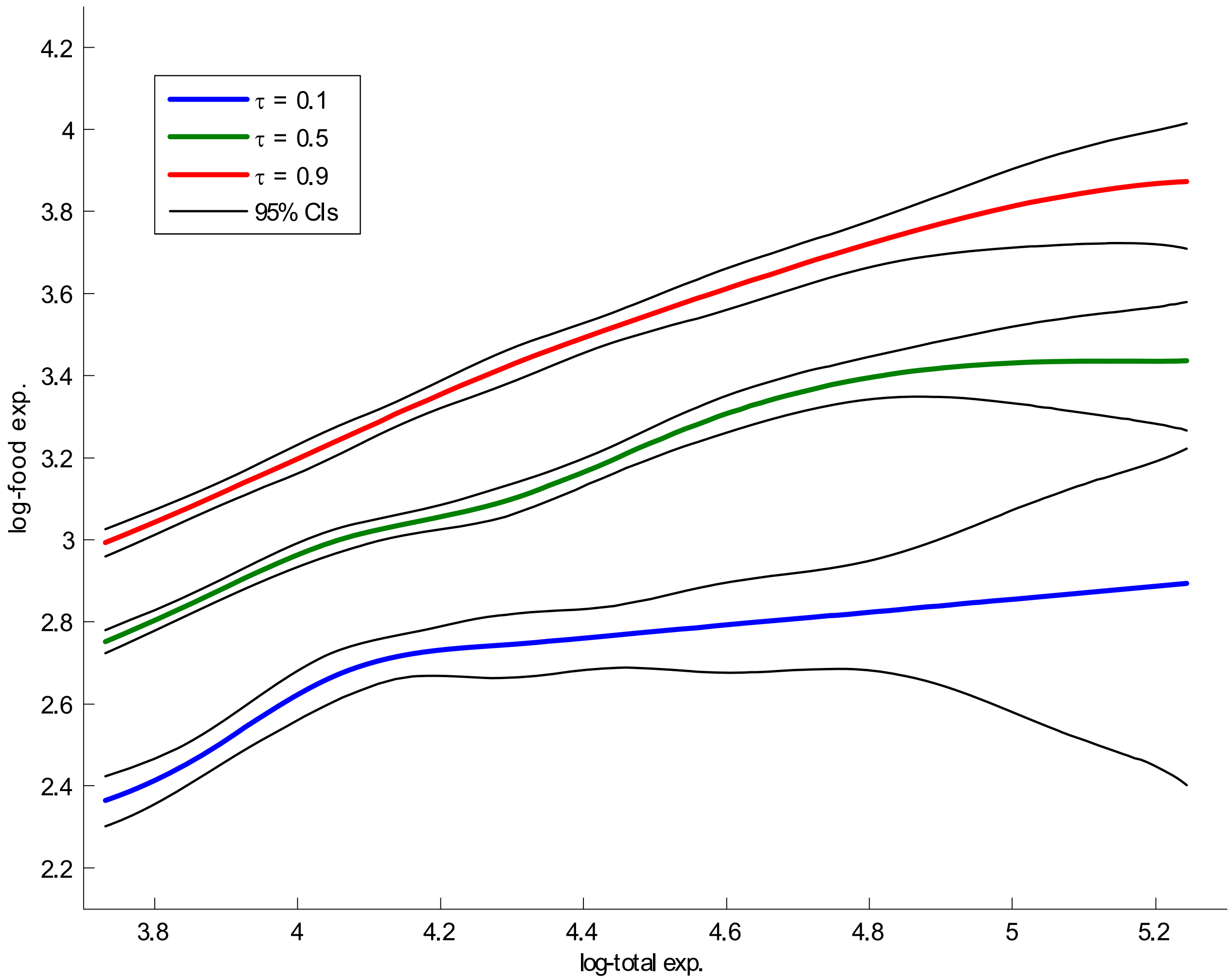


Figure 3a. Unrestrcited Quantile Expansion Paths: Food, 1986



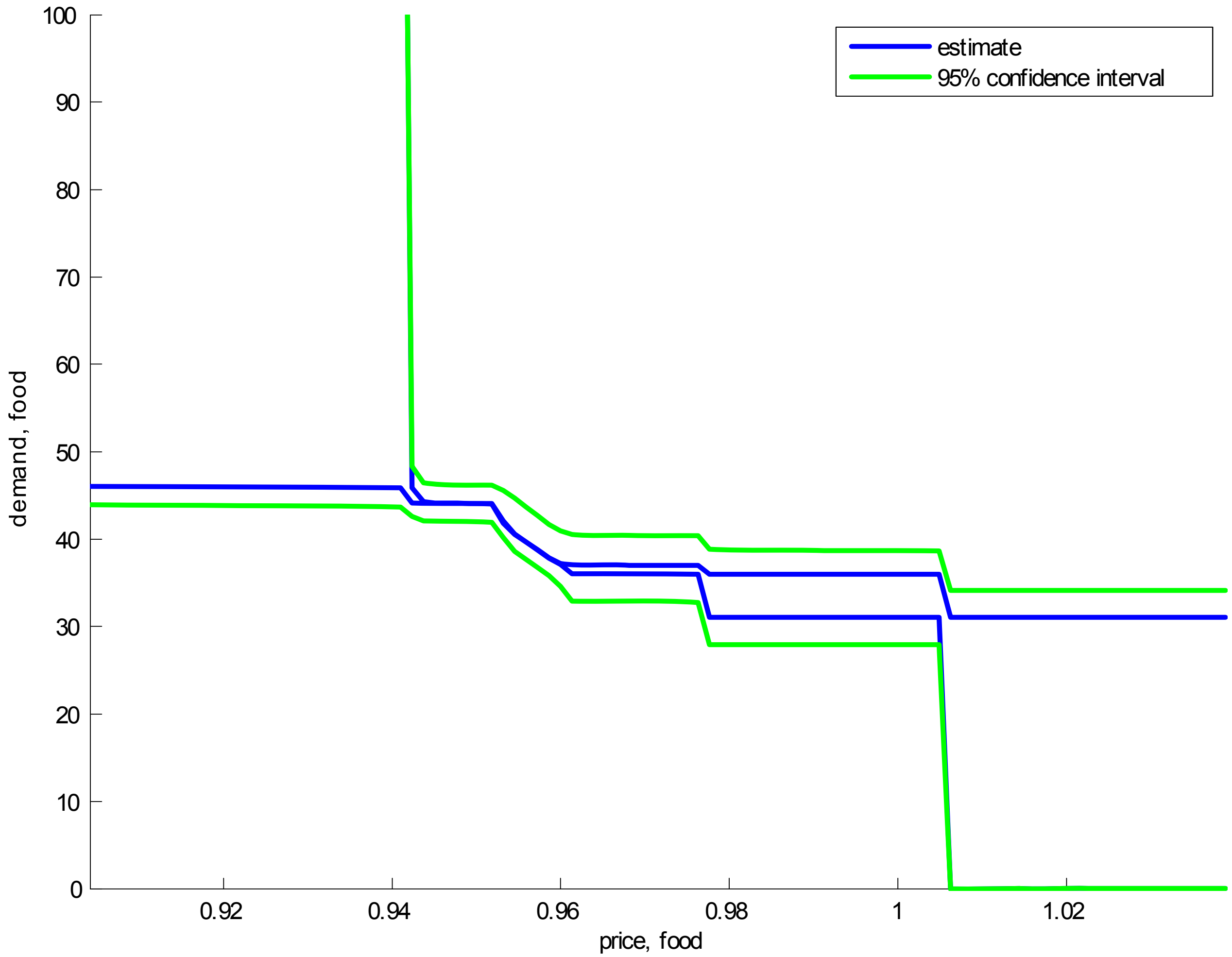
Blundell, Matzkin and Kristensen (2011)

Figure 3b. RP- Restrcited Quantile Expansion Paths: Food, 1986



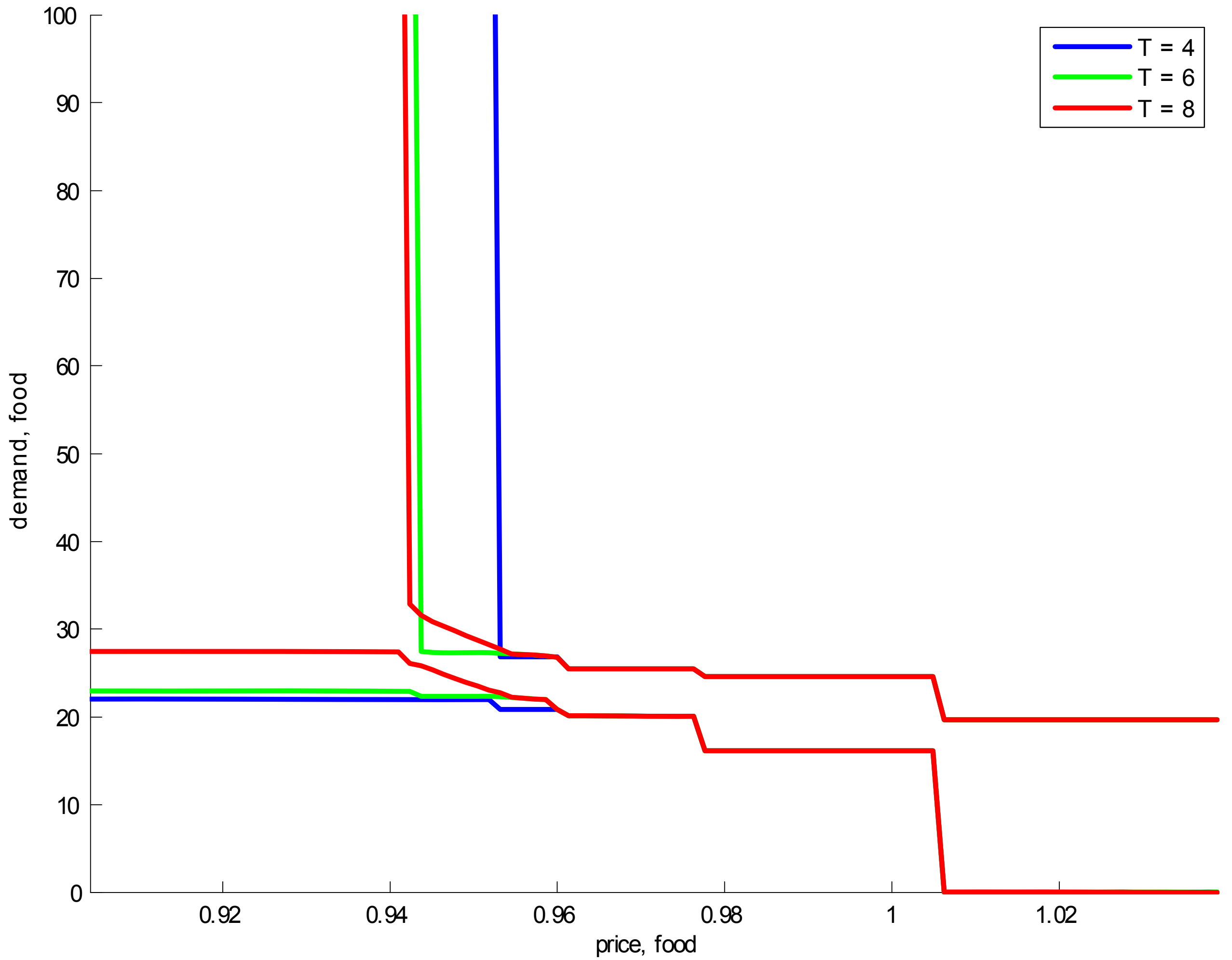
Blundell, Matzkin and Kristensen (2011)

Figure 4a: Quantile (RP-Restricted) Bounds on Demand (Median Income, τ=.5)



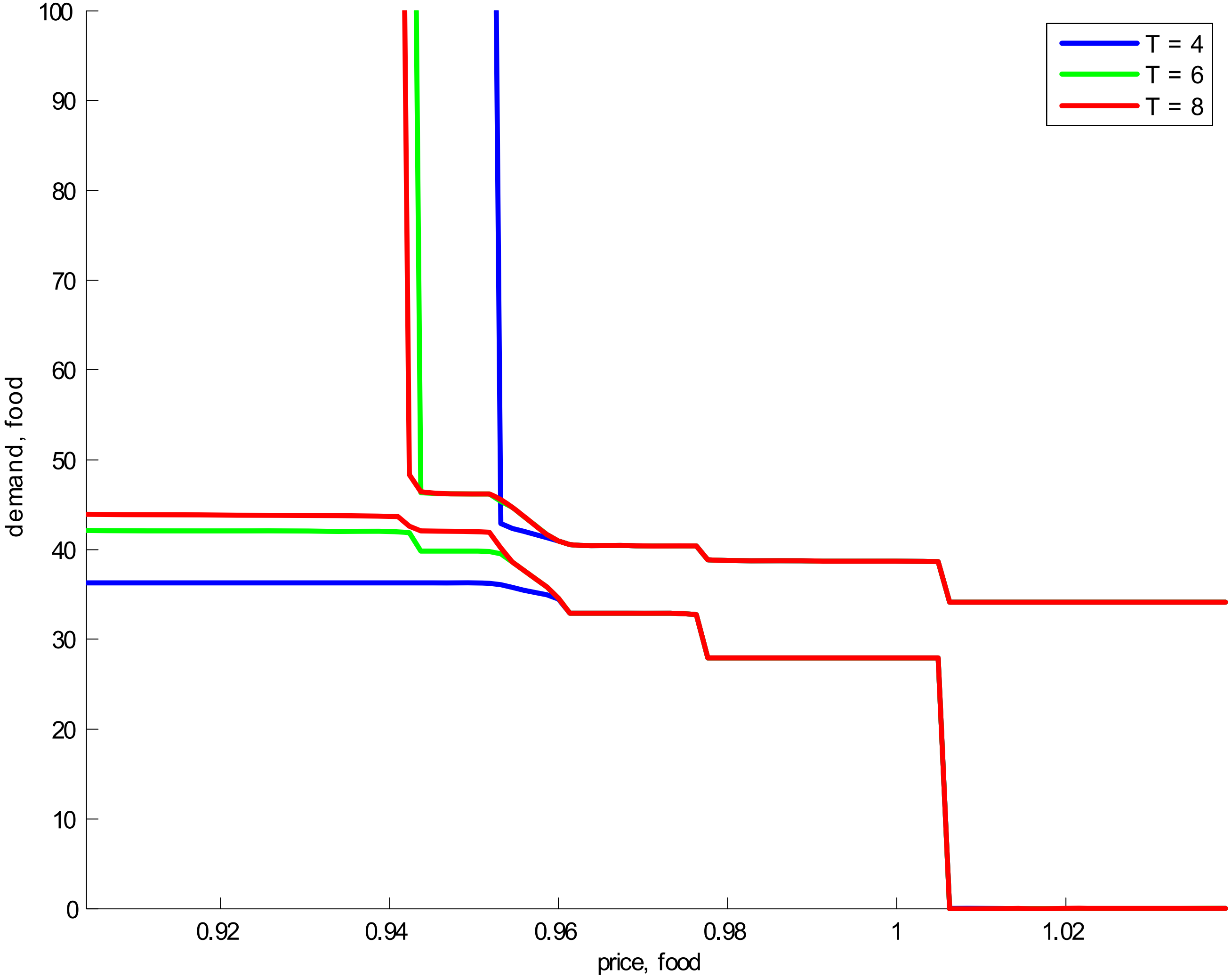
Blundell, Matzkin and Kristensen (2011)

Figure 4b: Quantile (RP-Restricted) Confidence Sets (Median Income, τ=.1)



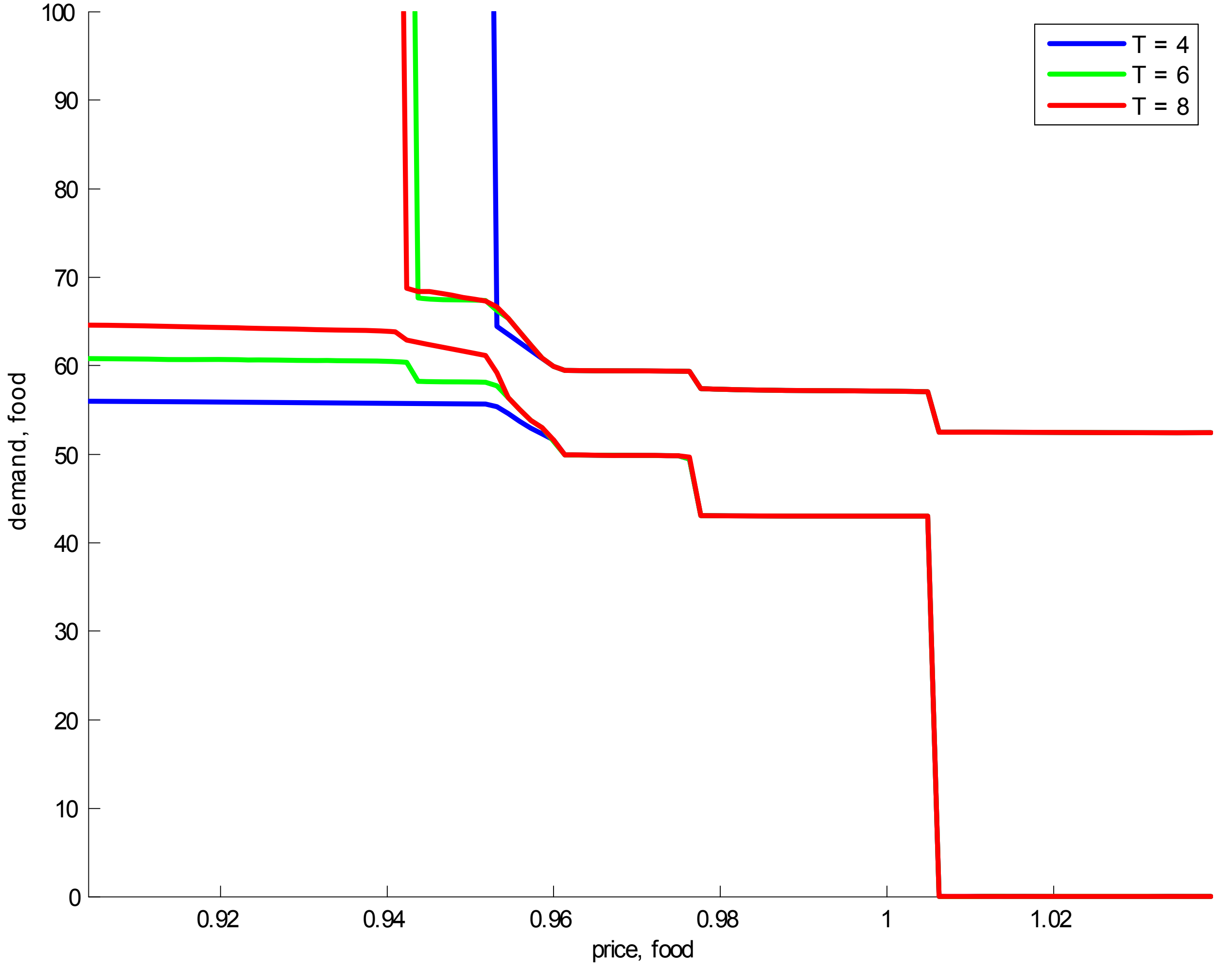
Blundell, Matzkin and Kristensen (2011)

Figure 4c: Quantile (RP-Restricted) Confidence Sets (Median Income, τ=.5)



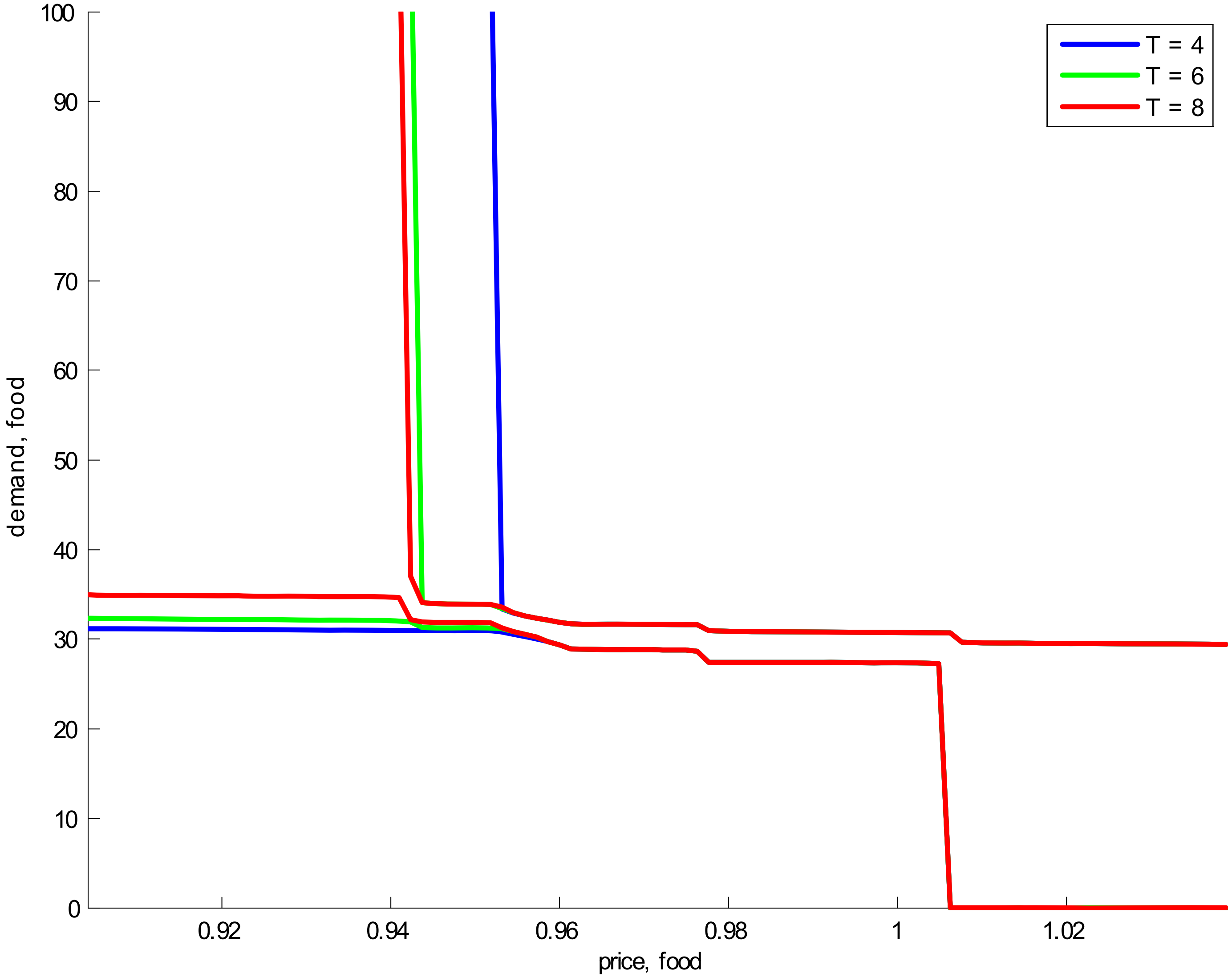
Blundell, Matzkin and Kristensen (2011)

Figure 4d: Quantile (RP-Restricted) Confidence Sets (Median Income, τ=.9)



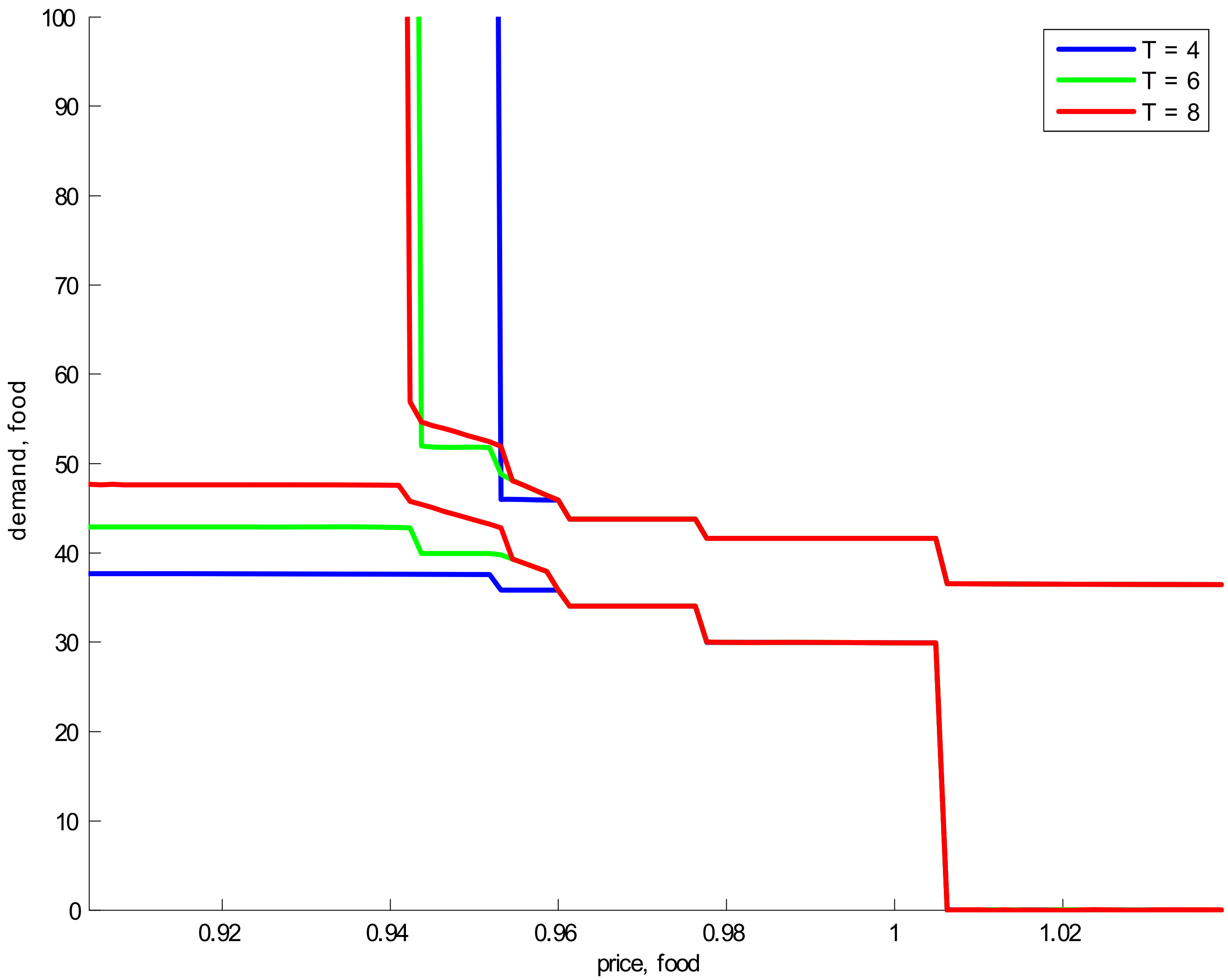
Blundell, Matzkin and Kristensen (2011)

Figure 4e: Quantile (RP-Restricted) Confidence Sets (25% Income, τ=.5)



Blundell, Matzkin and Kristensen (2011)

Figure 4f: Quantile (RP-Restricted) Confidence Sets (75% Income, τ=.5)



Blundell, Matzkin and Kristensen (2011)

Figure 5. Relative price data: 1975 to 1999 and price path

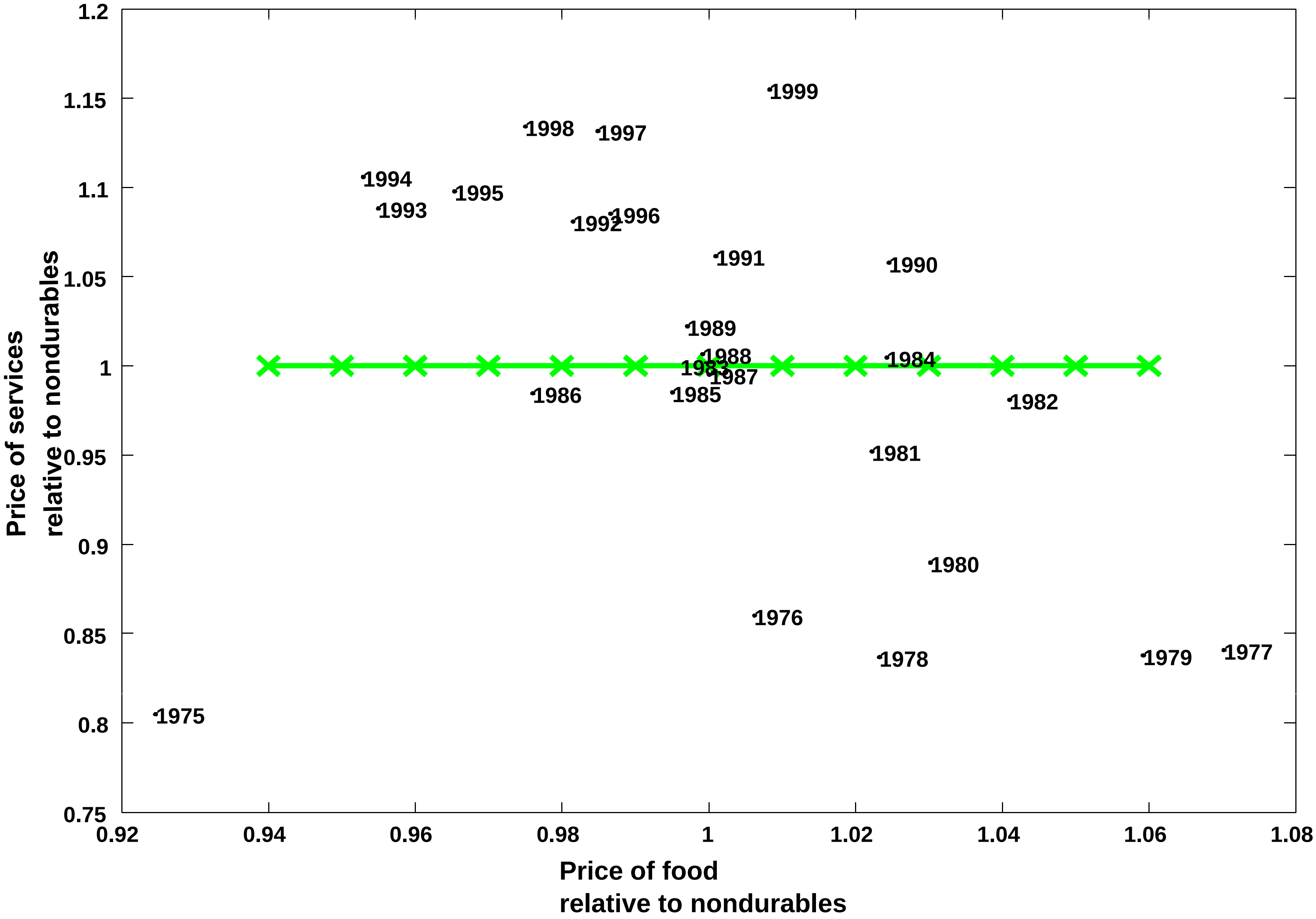
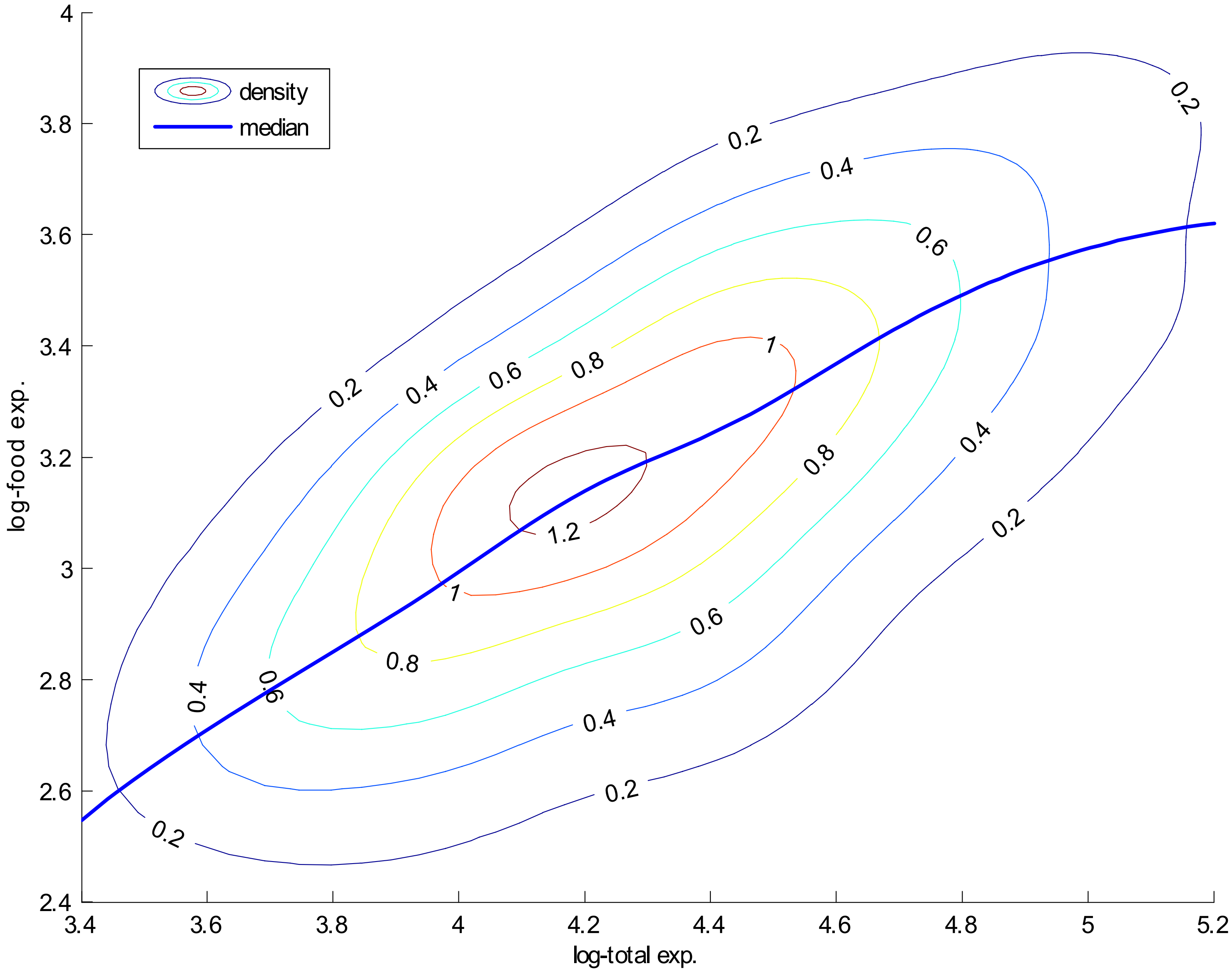


Figure 4a: Typical Joint Distribution of log food and log income



Blundell, Matzkin and Kristensen (2011)